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Exercise 2 - Numerical methods for fluid-structure interaction (Winter term 2015)

Exercise 1.1: The transformation $\hat{T} : \hat{\Omega} \to \Omega$ between different coordinate systems and domains requires to adapt the spatial derivative of a function f. Show that

$$\nabla f = \hat{\nabla} \hat{f} \hat{F}^{-1},$$

where $\hat{F}$ denotes the deformation gradient.

Exercise 1.2: The divergence of the Piola transformation is a key concept in solid mechanics and later in arbitrary Lagrangian-Eulerian (ALE) fluid-structure interaction. Let $\hat{\sigma} : \hat{\Omega} \to \mathbb{R}^3$ be the Piola transformation of $\sigma : \Omega \to \mathbb{R}^3$. Show that

$$\hat{\nabla} \cdot \hat{\sigma} = \hat{J} \nabla \cdot \sigma \quad \forall x = \hat{T}(\hat{x}), \hat{x} \in \hat{\Omega}.$$

Exercise 1.3: For the mathematical analysis and debugging it is often important to know how to simplify equations. Given the nonlinear elastodynamics equation

$$\hat{\rho}_s \partial_t^2 \hat{u} - \hat{\nabla} \cdot (\hat{F} \hat{\Sigma}) = \hat{\rho} \hat{f},$$

derive

$$-\nabla \cdot \Sigma = \rho f.$$

Which assumptions have been made in the approximation?

Discussion of exercises: Oct 19, 2015