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Exercise 3 - Numerical methods for fluid-structure interaction (Winter term 2015)

Exercise 3.1: Show that the do-nothing condition

$$\rho_f \nu \partial_n v - pn = 0 \quad \text{on } \Gamma_{out} \tag{0.0.1}$$

implicitly contains the pressure normalization

$$\int_{\Gamma_{out}} p\, ds = 0 \tag{0.0.2}$$

for a straight channel and Poiseuille flow.

Exercise 3.2: Let the following linear boundary value problem be given:

$$-\nabla \cdot (\widehat{\Sigma}_{lin}) = f \quad \text{in } \widehat{\Omega}, \tag{0.0.3}$$

$$\hat{u} = 0 \quad \text{on } \partial\widehat{\Omega}_D, \tag{0.0.4}$$

$$\widehat{\Sigma}_{lin}\hat{n} = \hat{g} \quad \text{on } \partial\widehat{\Omega}_N. \tag{0.0.5}$$

Show that finding a solution $\hat{u}$ of this strong form is formally equivalent to finding a solution of the corresponding weak formulation where we seek $\hat{u} \in \hat{V}_s^0 = \{\hat{u} \in H^1 | \hat{u} = 0 \text{ on } \partial\widehat{\Omega}_D\}$ such that

$$(\widehat{\Sigma}_{lin}, \widehat{\nabla}\hat{\varphi}) - \int_{\partial\hat{\Omega}_N} \widehat{\Sigma}_{lin} \cdot n\, \hat{\varphi}\, ds = (\hat{f}, \hat{\varphi}) \quad \forall \hat{\varphi} \in \hat{V}_s^0,$$

where

$$\widehat{\Sigma}_{lin} = 2\mu \widehat{E}_{lin} + \lambda \text{tr} \widehat{E}_{lin} \hat{I}.$$

Exercise 3.3:

Given

$$\hat{\rho}_s \partial_t \hat{v}_s - \widehat{\text{div}}(\widehat{F}\widehat{\Sigma}(\hat{u}_s)) + \gamma_w \hat{v}_s - \gamma_s \widehat{\text{div}}(\hat{\epsilon}(\hat{v}_s)) = 0 \quad \text{in } \widehat{\Omega}_s,\ t \in I,$$

$$\hat{\rho}_s(\partial_t \hat{u}_s - \hat{v}_s) = 0 \quad \text{in } \widehat{\Omega}_s,\ t \in I, \tag{0.0.6}$$

where $\hat{\epsilon}(\hat{v}_s)$ is defined by

$$\hat{\epsilon}(\hat{v}_s) = \frac{1}{2}(\widehat{\nabla}\hat{v}_s + \widehat{\nabla}\hat{v}_s^T),$$

and the damping parameters $\gamma_w \geq 0$ and $\gamma_s \geq 0$.

Show that

1. For a stress-free state $\widehat{F}\widehat{\Sigma}\hat{n}_s = 0$ on $\partial\widehat{\Omega}_s$ and (if $\gamma_s > 0$) $\hat{\epsilon}(\hat{v}_s)\hat{n}_s = 0$ on $\partial\widehat{\Omega}_s$, the following a priori energy estimate holds true:

$$\frac{d}{dt}\Big[\frac{\hat{\rho}_s}{2}\|\hat{v}_s\|^2_{\widehat{\Omega}_s} + \int_{\widehat{\Omega}_s} W(\widehat{E})\,d\hat{x}\Big] + \gamma_w\|\hat{v}_s\|^2_{\widehat{\Omega}_s} + \gamma_s||\hat{\epsilon}(\hat{v}_s)||^2_E = 0.$$

   That implies the following energy decay property:

$$E(t) = E(0) - \int_0^t \big[\gamma_w||\hat{v}_s(\tau)||^2_{\widehat{\Omega}_s} + \gamma_s||\hat{\epsilon}(\hat{v}_s)(\tau)||^2_E\big]\,d\tau,$$

   where

$$(\hat{\epsilon}(\hat{v}_s), \widehat{\nabla}\hat{v}_s)_{\widehat{\Omega}_s} =: ||\hat{\epsilon}(\hat{v}_s)||^2_E.$$

   and $\|\cdot\|_{\widehat{\Omega}_s}$ is the standard L^2 norm. Moroever, W is the hyperelastic energy with the property

$$\widehat{\Sigma}(\widehat{E}) = \frac{\partial W}{\partial \widehat{E}}(\widehat{E}).$$

2. What happens with the solid system for $\gamma_w, \gamma_s \to 0$ or $\gamma_w, \gamma_s \to \infty$?

3. What are physical and mathematical consequences when dealing with non-negative damping parameters γ_w and γ_s?

Discussion of exercises: Nov 9 / Nov 16, 2015