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Exercise 5 - Numerical methods for fluid-structure interaction (Winter term 2015)

Exercise 5.1: Proof the following theorem: Let $\widehat{\Omega}$ be a bounded domain with $C^{1,1}$-boundary. Furthermore, let $\hat{\mathcal{A}}$ be invertible in the closure of $\widehat{\Omega}$ and there holds for each $t \in I$ the two conditions

- $\Omega = \hat{\mathcal{A}}(\widehat{\Omega})$ is bounded and $\partial\Omega$ is Lipschitz-continuous.
- Let $\hat{\mathcal{A}} \in W^{1,\infty}(\widehat{\Omega})$ and $\hat{\mathcal{A}}^{-1} \in W^{1,\infty}(\Omega)$.

Then, $v \in H^1(\Omega)$ if and only if $\hat{v} = v \circ \hat{\mathcal{A}} \in H^1(\widehat{\Omega})$. Moreover, the corresponding norms are equivalent.

Exercise 5.2: This exercise is the extension of Exercise 3.3 including notation and definitions made therein. Given the fluid problem

$$\begin{aligned} \rho_f \hat{\partial}_t v_f + \rho_f (v_f - w)\cdot\nabla v_f - \nabla\cdot\sigma_f &= 0 \quad \text{in } \Omega_f(t),\ t \in I,\\ \nabla\cdot v &= 0 \quad \text{in } \Omega_f(t),\ t\in I, \end{aligned}$$

and the solid problem

$$\begin{aligned} \hat{\rho}_s \partial_t \hat{v}_s - \widehat{\operatorname{div}}(\widehat{F}\widehat{\Sigma}(\hat{u}_s)) + \gamma_w \hat{v}_s - \gamma_s \widehat{\operatorname{div}}(\hat{\epsilon}(\hat{v}_s)) &= 0 \quad \text{in } \widehat{\Omega}_s,\ t\in I,\\ \hat{\rho}_s(\partial_t \hat{u}_s - \hat{v}_s) &= 0 \quad \text{in } \widehat{\Omega}_s,\ t \in I, \end{aligned} \tag{0.0.1}$$

where

$$\sigma_f = -p_f I + 2\rho_f \nu_f D(v_f) = -p_f I + \rho_f\nu_f(\nabla v_f + \nabla v_f^T).$$

Proof the following theorem: Let the coupled fluid-structure problem be isolated, i.e., $v_f = 0$ on $\partial\Omega_f \setminus \Gamma_i$ and a stress-free state $\widehat{F}\widehat{\Sigma}\hat{n}_s = 0$ on $\partial\widehat{\Omega}_s \setminus \widehat{\Gamma}_i$ and (if $\gamma_s > 0$) $\hat{\epsilon}(\hat{v}_s)\hat{n}_s = 0$ on $\partial\widehat{\Omega}_s \setminus \widehat{\Gamma}_i$. Then, the following a priori energy estimate holds true:

$$\frac{d}{dt}\Big[\frac{\rho_f}{2}\|v_f\|^2_{\Omega_f} + \frac{\hat{\rho}_s}{2}\|\hat{v}_s\|^2_{\widehat{\Omega}_s} + \int_{\widehat{\Omega}_s} W(\widehat{E})\,\partial\hat{x}\Big] + 2\rho_f\nu_f\|D(v_f)\|^2_{\Omega_f} + \gamma_w\|\hat{v}_s\|^2_{\widehat{\Omega}_s} + \gamma_s||\hat{\epsilon}(\hat{v}_s)||^2_E = 0.$$

That implies the following energy decay property:

$$E(t) = E(0) - \int_0^t \big[\gamma_w||\hat{v}_s(\tau)||^2_{\widehat{\Omega}_s} + \gamma_s||\hat{\epsilon}(\hat{v}_s)(\tau)||^2_E + 2\rho_f\nu_f||D(v_f)(\tau)||^2_{\Omega_f}\big]\,d\tau.$$

Discussion of exercises: Nov 30, 2015