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Exercise 9 - Numerical methods for fluid-structure interaction (Winter term 2015)

Exercise 9.1:

Let the following nonlinear initial/boundary-value problem be given:
Find vector-valued displacements $u : \mathbb{R}^3 \to \mathbb{R}^3$ for all times $t \in [0,T]$ such that

$$\partial_t u - \nabla \cdot (F\Sigma) = f \quad \text{in } [0,T] \times \Omega, \tag{0.0.1}$$

$$u = 0 \quad \text{on } [0,T] \times \partial\Omega, \tag{0.0.2}$$

$$u = 0 \quad \text{in } \{t = 0\} \times \Omega. \tag{0.0.3}$$

Here, $F = I + \nabla u$ and $\Sigma = \mu \nabla u$ where I is the identity matrix and $\mu > 0$.

1. Derive the time-discretized scheme using a backward Euler discretization;
2. Derive the weak formulation on the spatially continuous level;
3. Next, apply a Galerkin FEM scheme (e.g., using Q_1^c elements) for spatial discretization;
4. Apply (formally) Newton's method to linearize the problem;
5. Write down the linear equation system that needs to be solved in each Newton step.

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Exercise 9.2:

Let the following stationary fluid-structure interaction problem be given:
Find $\{\hat{v}_f, \hat{u}_f, \hat{u}_s, \hat{p}_f, \hat{p}_s\} \in \{\hat{v}_f^D + \hat{V}^0_{f,\hat{v}}\} \times \{\hat{u}_f^D + \hat{V}^0_{f,\hat{u}}\} \times \{\hat{u}_s^D + \hat{V}^0_s\} \times \hat{L}^0_f$, such that

$$
\begin{aligned}
(\hat{\rho}_f \hat{J}(\widehat{F}^{-1}\hat{v}_f \cdot \widehat{\nabla})\hat{v}_f), \hat{\psi}^v)_{\widehat{\Omega}_f} & \\
+(\hat{J}\widehat{\sigma}_f \widehat{F}^{-T}, \widehat{\nabla}\hat{\psi}^v)_{\widehat{\Omega}_f} - \langle \hat{g}_f, \hat{\psi}^v \rangle_{\widehat{\Gamma}_N} - (\hat{\rho}_f \hat{J} \hat{f}_f, \hat{\psi}^v)_{\widehat{\Omega}_f} &= 0 \quad \forall \hat{\psi}^v \in \hat{V}^0_{f,\hat{v}}, \\
(\widehat{F}\widehat{\Sigma}, \widehat{\nabla}\hat{\psi}^v)_{\widehat{\Omega}_s} - (\hat{\rho}_s \hat{f}_s, \hat{\psi}^v)_{\widehat{\Omega}_s} &= 0 \quad \forall \hat{\psi}^v \in \hat{V}^0_s, \\
(\widehat{\sigma}_{\text{mesh}}, \widehat{\nabla}\hat{\psi}^u)_{\widehat{\Omega}_f} &= 0 \quad \forall \hat{\psi}^u \in \hat{V}^0_{f,\hat{u},\widehat{\Gamma}_i}, \\
(\widehat{\text{div}}\,(\hat{J}\widehat{F}^{-1}\hat{v}_f), \hat{\psi}^p)_{\widehat{\Omega}_f} &= 0 \quad \forall \hat{\psi}^p \in \hat{L}^0_f,
\end{aligned}
$$

with all quantities as defined in the lecture.

Tasks:

- Write the above problem in terms of a compact semi-linear form $\hat{A}(\hat{U})(\hat{\Psi}) = 0$;
- Linearize $A(\hat{U})(\hat{\Phi})$ in order to obtain the Jacobian matrix $A'(\hat{U})(\delta\hat{U}, \hat{\Psi})$.

Hint: For linearization the Gateaux derivative must be built for all search directions $\{\delta\hat{v}_f, \delta\hat{u}_f, \delta\hat{u}_s, \delta\hat{p}_f, \delta\hat{p}_s\}$.

Discussion of exercises: end of the class period