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Numerics of Partial Differential Equations – Tutorial 1

Home Assignment 1.1 [Recap: Numerical quadrature]

This exercise aims at recapitulating quadrature formulas for numerical solutions of integrals. Additionally, the relation to ordinary differential equations is studied.

(a) Let the function $g(x) := 2x\left(1+x^2\right)^{-2}$ be given. Approximate the integral $\int_0^4 g(x)dx$ using the Gauss-Legendre quadrature formula for $k=1$, by

(i) solving the problem through an affine transformation to the interval $[-1,1]$,

(ii) deriving the quadrature formula through an affine transformation to $[0,4]$ and applying it to g.

(b) Solve the following differential equation in integral formulation by integrating the left side exactly and integrating the right side with the trapezoidal rule

$$\int_{t_n}^{t_{n+1}} u'(t)dt = \int_{t_n}^{t_{n+1}} f(t,u(t))dt$$

Home Assignment 1.2 [Recap: Time-stepping schemes]

In this exercise the most important time-stepping schemes and their relationship to finite differences should be recapitulated. Let an ordinary differential equation (ODE) in abstract formulation

$$u'(t) = f(t,u)$$

be given. In this course we will denote the the time-step size $t_{n+1}-t_n$ by k or Δt.

(a) Which method arises, when starting from (t_n, u_n) the derivative is being replaced by the forward difference quotient?

(b) Which method arises, when starting from (t_{n+1}, u_{n+1}) the derivative is being replaced by the backward difference quotient?

Home Assignment 1.3 [Recap: Galerkin scheme for ODEs]

Important: This might yet be unknown to you, the essence of it is to understand the following:
If the equation has to hold for every φ then any conclusions we draw from choosing a specific $\hat{\varphi}$ hold in any case.

We consider the weak form of the abstract ODE formulation $[u'(t) = f(t,u)]$ on the interval $I = [t_0, t_0+T]$ which is given by

$$\int_I \langle u'(t) - f(t,u), \varphi\rangle dt = 0, \forall \varphi \in C(I)^d.$$

Try to understand that through a smart choice of a specific test function $\hat{\varphi} \in C(I)^d$ or a sequence of test functions, we can make deductions about the solution $u(t)$.
Here no proposition about the weak form should be derived, but only a logical consequence, which will be needed a few times throughout the semester.

This exercise sheet solely serves for the revision of topics from introductory numerical mathematics lectures, which will be used in this course. Questions can be asked in the first exercise, so please have a look at the assignments.