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Numerics of Partial Differential Equations – Tutorial 11

Exercise 11.1

Given is the time discretization of the heat equation by the implicit trapezoidal rule

$$Mu_h^n + \frac{1}{2}kAu_h^n = Mu_h^{n-1} - \frac{1}{2}kAu_h^{n-1}$$

with the matrices $M = \{(\varphi_i, \varphi_j)\}_{i,j=1}^M$ and $A = \{(\nabla\varphi_i, \nabla\varphi_j)\}_{i,j=1}^M$. Analyze the stability of this time discretization.

Home Assignment 11.2

Let A be an elliptic operator. The fractional-step-θ method with parameters θ, α, β and $\theta' = 1 - 2\theta$ is given by

$$u^{n+1} = (I + \alpha\theta kA)^{-1}(I - \beta\theta kA)(I + \beta\theta' kA)^{-1}(I - \alpha\theta' kA)(I + \alpha\theta kA)^{-1}(I - \beta\theta kA)u^n.$$

(a) Derive the stability function $R(z)$.

(b) Show that for the stability function

$$\lim_{z\to\infty} |R(z)| = \frac{\beta}{\alpha}$$

holds. Therefore, $\alpha \geq \beta$ must hold to achieve stability for the greatest eigenvalue of A.